E 2384 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2015

Second Semester

Complementary Mathematics—INTEGRAL CALCULUS AND MATRICES

(Complementary Course to Physics/Chemistry/Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[Prior to 2013 Admissions]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
A bunch of 4 questions has weight 1.

I. 1 Express $\lim_{\|P\| \to 0} \sum_{k=1}^{n} (3\,C_k^{\,2} - 2\,C_k + 5)\,\Delta x_k$ as an integral if P denote a partition of [–1, 3].

2 If f is integrable on $[a, b]$, then what is the average value of f on $[a, b]$.

3 If f is continuous and that $\int_0^3 f(z)\,dz = 3$ and $\int_0^4 f(z)\,dz = 7$. Find $\int_3^4 f(z)\,dz$.

4 Find $\frac{dy}{dx}$ if $y = \int_1^x \frac{1}{t}\,dt,\ x > 0$.

II. 5 State fundamental theorem of integral calculus.

6 The region between the curve $y = \sqrt{x}$ $0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find the volume of the solid ?

7 What is a smooth curve ?

8 If the function $f(x) \ge 0$ is smooth on $[a, b]$. Write the formula for the area of the surface generated by revolving the curve $y = f(x)$ about the x-axis.

III. 9 State Fubini's theorem in the first form.

10 Find the area of the region bounded by $y = x$ and $y = x^2$ in the first quadrant.

11 Evaluate : $\int_0^{\pi} \int_0^{\sin x} y\,dy\,dx$.

12 Sketch the region of integration of $\int_0^3 \int_0^2 (4 - y^2)\,dy\,dx$.

Turn over

IV. 13 If the rank of a matrix is 3, what is the rank of its transpose ?

14 What is a matrix polynomial ?

15 State Cayley-Hamilton theorem.

16 The eigen values of a real symmetric matrix are ———.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Graph the integrand and use area to evaluate the integral $\int_{-2}^{4}\left(\frac{x}{2}+3\right)dx$.

18 Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

19 Find the area of the region enclosed by $y = x^4$ and $y = 8x$.

20 A pyramid 3m high as a square base that is 3m on a side. The cross-section of the pyramid perpendicular to the altitude x_m down from the vertex is a square x_m on a side. Find the volume of the pyramid.

21 Evaluate the integral :

$$\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dy\,dx.$$

22 Evaluate : $\iint_R e^{x^2+y^2}\,dy\,dx$, where R is the semi circular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

23 Find the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$.

24 Using Cayley-Hamilton theorem find A^3 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 What values of a and b maximize that value of $\int_a^b (x - x^2)\, dx$.

26 Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$ and the curve $y = \frac{1}{x^2}$ and the x-axis.

27 Find the length of $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.

28 Evaluate : $\iint_{12} xy\, d\,\mathrm{A}$ where R is the region bounded by the lines $y = x$, $y = 2x$ and $x + y = 2$.

29 Find the area of the region cut from the first quadrant by the curve $r = 2\,(2 - \sin 2\theta)^{1/2}$.

30 Find the inverse of the matrix A and hence solve the equation AX = Y, where $\mathrm{A} = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$,

$\mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathrm{Y} = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Find the volume of the solid generated by revolving the region in the first quadrant bounded above the line $y = 2$, below by the curve $y = 2 \sin x$, $0 \le x \le \pi/2$ and on the left by y-axis, about the line $y = 2$.

32 Find the average value of $F(x, y, z) = xyz$ over the cube bounded by the co-ordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant.

33 Find the eigen values and corresponding eigen vectors of $\begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$.

$(2 \times 4 = 8)$