E 5169 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II Mathematics and Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define Supremum of a set.
2. Find the supremum and infimum of the set $\left\{1+\frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$.
3. Give an example of a subset of R that is neither open nor closed.
4. Give an example to show that arbitrary intersection of open sets need not be open.
5. Define convergence of a sequence of real numbers.
6. Give an example of a sequence which oscillates infinitely.
7. Define Cauchy sequence.
8. Express $\frac{-1+3i}{2-i}$ in the form $a + ib$.
9. Express $-1-i$ in the exponential form.
10. Give an example of an open connected set in the complex plane.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find the supremum and infimum of the set $S = \{x \in R : 3x^2 - 10x + 3 < 0\}$.
12. Show that the supremum of a set if it exists is unique.
13. State Dedekind's property.
14. Illustrate by an example that union of an arbitrary family of closed sets may not be a closed set.
15. Show that the set of all integers is countable.

Turn over

16. Prove that the intersection of two open subsets of R is open in R.

17. Show that the sequence $\{S_n\}$ where $S_n = 1+\frac{1}{2}+\ldots+\frac{1}{n}$ cannot converge.

18. Show that $\lim \frac{1+2+\ldots+n}{n^2} = \frac{1}{n}$.

19. Give an example of two divergent sequences so that their product converges.

20. Sketch the following set and determine whether it is a domain $1\ mz > 1$.

21. Sketch the set of points determined by the condition Re $(\bar{z} - i) = 2$.

22. Reduce $\frac{5i}{(1-i)(2-i)(3-i)}$ into a real number.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. State the properties of the set of real numbers as a complete ordered field.

24. State and prove the Archimedean property of real numbers.

25. Show that every open set is the union of open intervals.

26. Show that the derived set of a bounded set is bounded.

27. Use Cauchy's general principle of convergence to show that $\left\{\frac{(-1)^n}{n}\right\}$ converges.

28. Show that $\lim a_n = a$ implies that $\lim |a_n| = |a|$. Is the converse true. Justify.

29. Show that $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right] = 1$.

30. Find all the roots in rectangular co-ordinates of $(-16)^{1/4}$.

31. By writing the individual factors on the left in exponential form show that $i(1-\sqrt{3}i)(\sqrt{3}+i) = 2(1+\sqrt{3}i)$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Prove that the set of real numbers in [0, 1] is uncountable.

    (b) Prove that a set is closed if and only if its complement is open.

33. (a) Prove that every infinite bounded set has a limit point.

    (b) Prove that the union of an arbitrary family of open sets is open.

34. Prove that every convergent sequence is bounded and has a unique limit.

35. (a) Show that $\{S_n\}$, where $S_n = \left(1+\frac{1}{n}\right)^n$ is convergent and its limit lies between 2 and 3.

    (b) Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

$(2 \times 15 = 30)$