E 5056 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2013

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(For Model I, Model II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
A bunch of **four** *questions has weight* 1.

I. 1. The additive inverse of the zero vector is ———.

2. State whether true or false : $S = \{[a, b] \in \mathbb{R}^2 : a + b = 1\}$ is a subspace of $\mathbb{R}^2$, considered as row matrices.

3. If $\{u\}$ is a linearly dependent subset of a vector space V, then $u =$ ———.

4. What is the dimension of the vector space $\mathbb{P}^n$?

II. 5. The row space of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is ———.

6. If $T : \mathbb{P}^2 \to \mathbb{M}_{2\times 2}$ is defined by $T(at^2 + bt + c) = \begin{bmatrix} a & 2b \\ 0 & a \end{bmatrix}$ for $a, b, c \in \mathbb{R}$, then what is $T(t)$?

7. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation with $T[1 \;\; 0] = [1 \;\; 2 \;\; 0]$ and $T[0 \;\; 1] = [0 \;\; 3 \;\; 4]$, then $T[1 \;\; 1] =$ ———.

8. Define null space of a linear transformation.

III. 9. Give the matrix representation of $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T[a \;\; b] = [a \;\; b]$ with respect to the standard basis for $\mathbb{R}^2$, considered as row matrices.

10. If a linear transformation $T : V \to W$ is one to one then nullity (T) = ———.

11. Define the discrete metric on an arbitrary non-empty set X.

12. What is the usual metric on the complex plane $\mathbb{C}$?

Turn over

IV. 13. Consider the real line $\mathbb{R}$ with the usual metric. Is it true that $\{\alpha\}$ is a closed set for any α in $\mathbb{R}$?

14. Let $A = \{z : |z| < 1\}$ be a subset of the complex plane. What is its boundary ?

15. Give an example of a metric space which is not complete ?

16. Let X, Y be metric spaces. Give a characterization of a continuous mapping from X to Y in terms of convergence of sequences.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17. Determine whether $S = \left\{\begin{bmatrix} x & y & z \end{bmatrix} \in \mathbb{R}^3 : y = 0\right\}$ is a subspace of $\mathbb{R}^3$.

18. Define a basis for a vector space. Write the standard basis for $\mathbb{M}_{2\times 3}$.

19. Define a linear transformation. If V is a vector space show that the transformation $T : V \to V$ defined by $T(v) = kv$ for all v is V and any scalar k is linear.

20. Show that the kernel of a linear transformation is a subspace of the domain.

21. Show by an example that the intersection of an infinite number of open sets in a metric space need not be open.

22. Let X be a metric space. If a subset F of X is closed then prove that its complement F^c is open.

23. Let X be a metric space. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X such that $x_n \to x$ and $y_n \to y$. Show that $d(x_n, y_n) \to d(x, y)$.

24. Let X, Y be metric spaces. Show that a constant mapping f from X into Y is continuous.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25. Determine whether the set $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is a basis for $\mathbb{R}^2$, considered as column vectors.

26. Determine the co-ordinate representation of the matrix $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ with respect to the basis

$$B = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}.$$

27. Find the matrix representation with respect to the standard basis for $\mathbb{R}^2$ and the standard basis $C = \{t^2, t, 1\}$ for $\mathbf{P}^2$ for the linear transformation $T : \mathbb{R}^2 \to \mathbf{P}^2$ defined by

$$T\begin{bmatrix} a \\ b \end{bmatrix} = 2at^2 + (a+b)t + 3b.$$

28. Determine whether the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$ defined by $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ a-b \\ 2a+3b \end{bmatrix}$ is one to one.

29. Let X be a metric space with metric d. Show that d_1 defined by $d_1(x, y) = d(x, y) / (1 + d(x, y))$ is also a metric on X.

30. Let X, Y be metric spaces. If $f : X \to Y$ is continuous then prove that $f^{-1}(G)$ is open in X wherever G is open in Y.

$(4 \times 2 = 8)$

Part D

Answer any two questions.
Each question has weight 4.

31. State and prove the theorem which provides a complete geometric interpretation of subspaces in $\mathbf{R}^3$.

32. If T is a linear transformation from an n-dimensional vector space V into W, prove that rank (T) + nullity (T) = dim (V).

33. If X is a complete metric space and Y is a subspace of X, then prove that Y is complete if and only if Y is closed.

$(2 \times 4 = 8)$